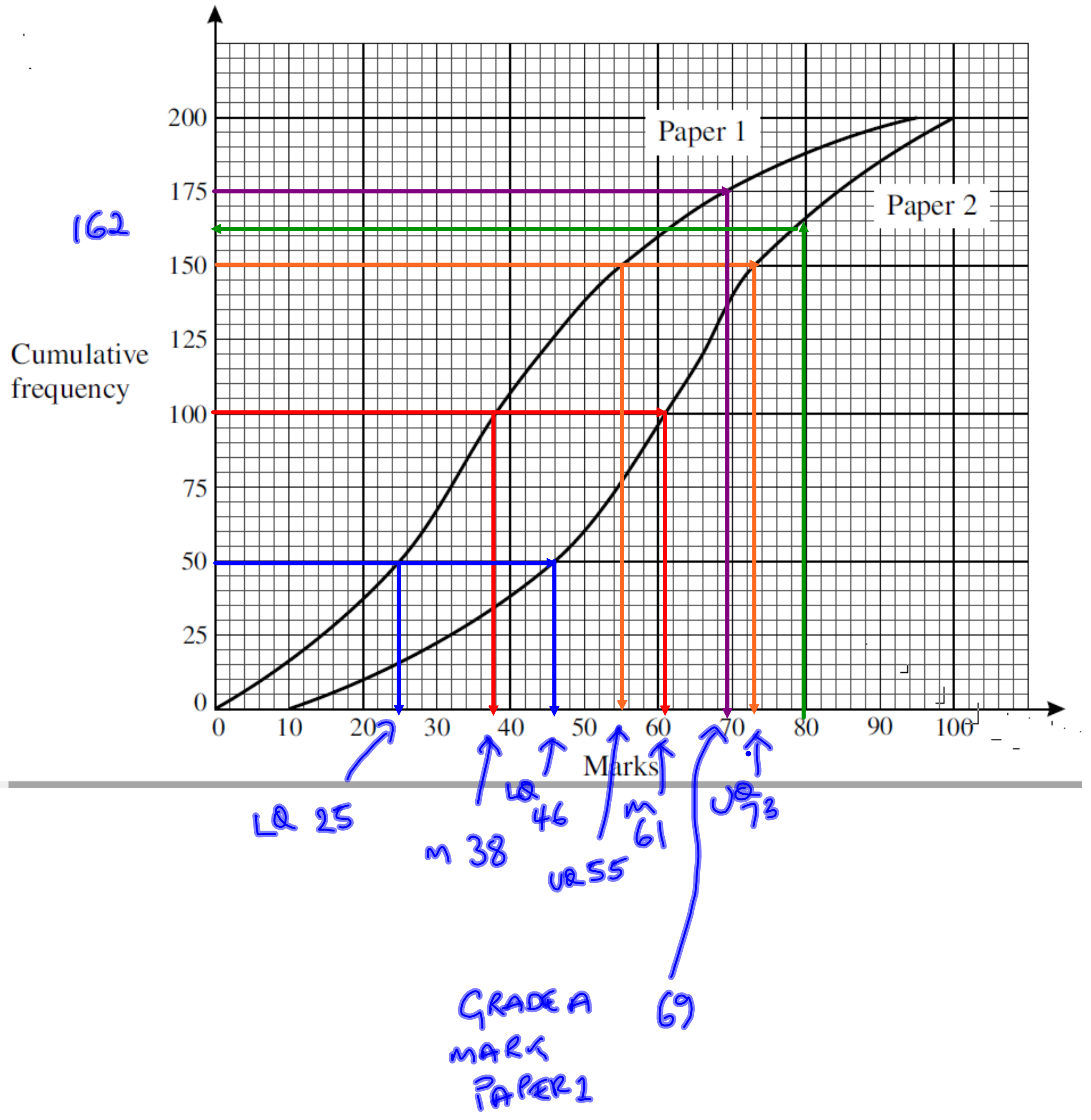
JANUARY 2011

1 200 candidates took each of two examination papers. The diagram shows the cumulative frequency graphs for their marks.



(i) Estimate the median mark for each of the papers. **[2]**

1 (i) Paper 1 38 ✓
Paper 2 61 ✓

2

(ii) State, with a reason, which of the two papers was the easier one. **[2]**

(ii) Paper 2 ✓ was easier because the median ✓ mark was higher

2

(iii) It is suggested that the marks on Paper 2 were less varied than those on Paper 1. Use interquartile ranges to comment on this suggestion. **[4]**

(iii)

	UQ	LQ	IQR
Paper 1	55 ✓	25 ✓	30 ✓
Paper 2	73 ✓	46 ✓	27 ✓

4

The IQR for paper 2 was 27 compared with 30 for paper 1. This is slightly less variation in marks but not highly significant.

✓

(iv) The minimum mark for grade A, the top grade, on Paper 1 was 10 marks lower than the minimum mark for grade A on Paper 2. Given that 25 candidates gained grade A in Paper 1, find the number of candidates who gained grade A in Paper 2. **[2]**

(iv) mark for Grade A on paper 1 is 69
mark for Grade A on paper 2 is 79
162 students got 79 or less on paper 2
So 38 students got A grades. ✓ MS 37 ± 3

2

(v) The mean and standard deviation of the marks on Paper 1 were 36.5 and 28.2 respectively. Later, a marking error was discovered and it was decided to add 1 mark to each of the 200 marks on Paper 1. State the mean and standard deviation of the new marks on Paper 1. **[2]**

(v) mean 37.5 ✓
standard deviation 28.2 ✓

2

2 The random variable X has the distribution Geo(0.2). Find

(i) $P(X = 3)$, [2]

2 $X \sim Geo(0.2)$

(i) $P(X=3) = 0.8^2 \times 0.2$

$= 0.128$ ✓

2

(ii) $P(3 \leqslant X \leqslant 5)$, [3]

(ii) $P(3 \leq X \leq 5) = P(X=3) + P(X=4) + P(X=5)$

$= 0.128 + (0.8^3 \times 0.2) + (0.8^4 \times 0.2)$

$= 0.31232$

$= 0.312$ (3sf) ✓

3

(iii) P($X > 4$). **[3]**

(iii) $P(X>4)$ is the probability of at least 4 failures

$P(X>4) = 0.8^4$

$= 0.4096$

$= 0.410$ (3sf)

3

Two independent values of X are found.

(iv) Find the probability that the total of these two values is 3. **[3]**

(iv) $P(\text{total } 3) = (P(X=1) \times P(X=2)) + (P(X=2) \times P(X=1))$

3

$P(X=1) = 0.2$

$P(X=2) = 0.8 \times 0.2 = 0.16$

$P(\text{total } 3) = (0.2 \times 0.16) + (0.16 \times 0.2)$

$= 0.032 + 0.032$

$= 0.064$

3 A firm wishes to assess whether there is a linear relationship between the annual amount spent on advertising, £x thousand, and the annual profit, £y thousand. A summary of the figures for 12 years is as follows.

$n = 12 \quad \Sigma x = 86.6 \quad \Sigma y = 943.8 \quad \Sigma x^2 = 658.76 \quad \Sigma y^2 = 83\,663.00 \quad \Sigma xy = 7351.12$

(i) Calculate the product moment correlation coefficient, showing that it is greater than 0.9. **[3]**

$$r = \frac{S_{xy}}{\sqrt{S_{xx} S_{yy}}}$$

$$S_{xy} = \Sigma xy - \frac{\Sigma x \Sigma y}{n}$$

$$= 7351.12 - \frac{86.6 \times 943.8}{12}$$

$$= 540.03$$

$$S_{xx} = \Sigma x^2 - \frac{(\Sigma x)^2}{n}$$

$$= 658.76 - \frac{(86.6)^2}{12}$$

$$= 33.79\dot{6}$$

$$S_{yy} = \Sigma y^2 - \frac{(\Sigma y)^2}{n}$$

$$= 83663 - \frac{943.8^2}{12}$$

$$= 9433.13$$

$$r = \frac{540.03}{\sqrt{33.79\dot{6} \times 9433.13}}$$

$$= 0.9564296754 86$$

$$= 0.956 \text{ (3sf)}$$

3

(ii) Comment briefly on this value in this context. **[1]**

This high positive value of r (close to 1) indicates a very strong link between the amount of profit made and the amount of money spent on advertising ✓

1

(iii) A manager claims that this result shows that spending more money on advertising in the future will result in greater profits. Make two criticisms of this claim. **[2]**

This strong relationship is apparent over the range of values used for advertising to date. You cannot extrapolate from this data to predict this relationship will hold over a wider range. ✓

MARK SCHEME:

Relationship may not continue

or Can't extrapolate

or Any indication that pattern may not continue

or Must state or imply referring to future

Although there is a strong correlation between the two values this does not imply that one causes the ~~other~~ to increase. ✓

MARK SCHEME:

Corr'n not imply causation

or Increase in profit may not be due to increase in spend on advertising.

or Variables may be increasing separately

2

(iv) Calculate the equation of the regression line of y on x. **[4]**

$$b = \frac{S_{xy}}{S_{xx}} = \frac{540.03}{33.79\dot{6}}$$

$$= 15.97879475$$

$$= 16.0 \text{ (3sf)}$$

$$a = \bar{y} - b\bar{x}$$

$$\bar{y} = \frac{\Sigma y}{n} = \frac{943.8}{12}$$

$$= 78.65$$

$$\bar{x} = \frac{\Sigma x}{n} = \frac{86.6}{12} = 7.21\dot{6}$$

$$a = 78.65 - (16.\dot{0} \times 7.21\dot{6})$$

$$= -36.66863546$$

$$= -36.7 \text{ (3sf)}$$

$$y = -36.7 + 16x$$

4

(v) Estimate the annual profit during a year when £7400 was spent on advertising. **[2]**

$y = -36.7 + 16x$

$= -36.7 + (16 \times 7.4)$

$= 81.7$ thousand

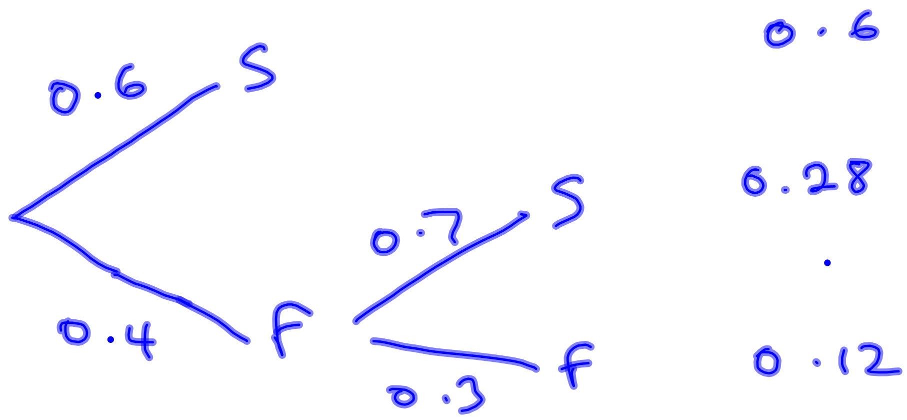
$= £81,700$ ✓

2

MS: 81,400 → 81,750

4 Jenny and Omar are each allowed two attempts at a high jump.

(i) The probability that Jenny will succeed on her first attempt is 0.6. If she fails on her first attempt, the probability that she will succeed on her second attempt is 0.7. Calculate the probability that Jenny will succeed. **[3]**

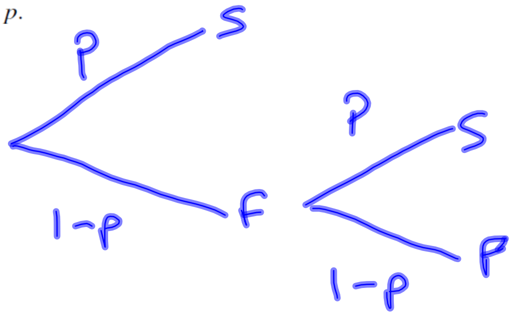


$P(\text{success}) = 0.6 + 0.28$

$= \underline{0.88}$ ✓

3

(ii) The probability that Omar will succeed on his first attempt is p. If he fails on his first attempt, the probability that he will succeed on his second attempt is also p. The probability that he succeeds is 0.51. Find p. **[4]**



$$P(\text{success}) = p + p(1-p) = 0.51$$

$$p + p - p^2 = 0.51$$

$$p^2 - 2p + 0.51 = 0$$

Factor pairs of 51

$3 \times 17 = 51$

Factor pairs of 0.51

$0.3 \times 1.7 = 0.5$

$(p-0.3)(p-1.7) = 0$

so $p = 0.3$ or $p = 1.7$

as it is a probability (ie between 0 and 1)

$\underline{p = 0.3}$ ✓

4

or solve using QF

$$p = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{2 \pm \sqrt{4 - (4 \times 0.51)}}{2}$$

$$= \frac{2 \pm \sqrt{4 - 2.04}}{2}$$

$$= \frac{2 \pm \sqrt{1.96}}{2}$$

$$p = \frac{2 \pm 1.4}{2}$$

$$p = \frac{2 + 1.4}{2} = 1.7$$

$$p = \frac{2 - 1.4}{2} = 0.3$$

$\underline{p = 0.3}$

(not $p = 1.7$ because $1.7 > 1$)

5 30% of packets of Natural Crunch Crisps contain a free gift. Jan buys 5 packets each week.

(i) The number of free gifts that Jan receives in a week is denoted by X. Name a suitable probability distribution with which to model X, giving the value(s) of any parameter(s). State any assumption(s) necessary for the distribution to be a valid model. **[4]**

Binomial ✓ 1 $X \sim B(5, 0.3)$ ✓ 1

The probability that any packet contains a gift is independent. ✓ 1

The probability that any packet contains a gift is constant. ✓ 1

Assume now that your model is valid.

(ii) Find

$X \sim B(5, 0.3)$

(a) P($X \leqslant 2$), **[1]**

$P(X \leq 2) = 0.8369$ (from table)

$= 0.837$ (3sf)

1

(b) P($X = 2$). **[2]**

$P(X=2) = \binom{5}{2} \times 0.3^2 \times 0.7^3$

$= \frac{5!}{3!2!} \times 0.3^2 \times 0.7^3$

$= \frac{5 \times 4 \times \cancel{3} \times \cancel{2} \times \cancel{1}}{\cancel{3} \times 2 \times \cancel{1} \times \cancel{2} \times \cancel{1}} \times 0.3^2 \times 0.7^3$

$= 10 \times 0.3^2 \times 0.7^3 = 0.3087$

$= 0.309$ (3sf)

2

(iii) Find the probability that, in the next 7 weeks, there are exactly 3 weeks in which Jan receives exactly 2 free gifts. **[3]**

$Y \sim B(7, 0.308)$

$P(Y=3) = \binom{7}{3} \times 0.308^3 \times 0.692^4$

$\frac{7 \times \cancel{6} \times 5 \times \cancel{4} \times \cancel{3 \times 2 \times 1}}{\cancel{4 \times 3 \times 2 \times 1} \times \cancel{3 \times 2 \times 1}}$

$= 35 \times 0.308^3 \times 0.692^4$

$= 0.2345$

$= \underline{0.235}$ (3sf)

3

6 (i) The diagram shows 7 cards, each with a digit printed on it. The digits form a 7-digit number.

1	3	3	3	5	5	9

How many different 7-digit numbers can be formed using these cards? [3]

$$\frac{7!}{3!\,2!}$$

Number	Freq
1	1
3	3
5	2
9	1

divide by 3!2! to remove duplicate numbers

$$= \frac{7\times6\times5\times\overset{2}{\cancel{4}}\times\cancel{3\times2\times1}}{\cancel{3\times2\times1}\times\cancel{2\times1}}$$

$$= 7\times6\times5\times2$$

$$= 42\times10$$

$$= \underline{\underline{420}}$$ ✓

3

(ii) The diagram below shows 5 white cards and 10 grey cards, each with a letter printed on it.

A	C	E	J	T
A	B	G	L	M
N	P	Q	R	Z

From these cards, 3 white cards and 4 grey cards are selected at random **without** regard to order.

(a) How many selections of seven cards are possible? [3]

WHITE CARDS $\binom{5}{3} = 5C3 = \dfrac{5\times\overset{2}{\cancel{4}}\times\cancel{3\times2\times1}}{\cancel{3\times2\times1}\times\cancel{2\times1}} = 10$

GREY CARDS $\binom{10}{4} = 10C4 = \dfrac{10\times\overset{3}{\cancel{9}}\times\cancel{8}\times7\times\cancel{6\times5}\times\cancel{4\times3\times2\times1}}{\cancel{6\times5}\times\cancel{4}\times\cancel{3}\times\cancel{2}\times1\times\cancel{4\times3\times2\times1}}$

$= 210$

No. of selections $= 210\times10$

$= 2100$ ✓

3

(b) Find the probability that the seven cards include exactly one card showing the letter A. **[4]**

Number of ways of choosing A from white.

$\binom{5}{3} = 10$

ACE
ACJ
ACT
AEJ
AET
AJT
CEJ
CET
CJT
EJT

A and 2 others

A $\underline{X}$ $\underline{X}$

$\binom{4}{2} = \frac{4 \times 3 \times \cancel{2 \times 1}}{2 \times 1 \times \cancel{2 \times 1}}$

$= \frac{12}{2} = 6$

P(A from white) $= \frac{6}{10} = \frac{3}{5}$

NUMBER OF WHITE FROM GREY

$\binom{10}{4} = 210$

A $\underline{X}$ $\underline{X}$ $\underline{X}$

A and 3 others

$\binom{9}{3} = \frac{9!}{6!.3!}$

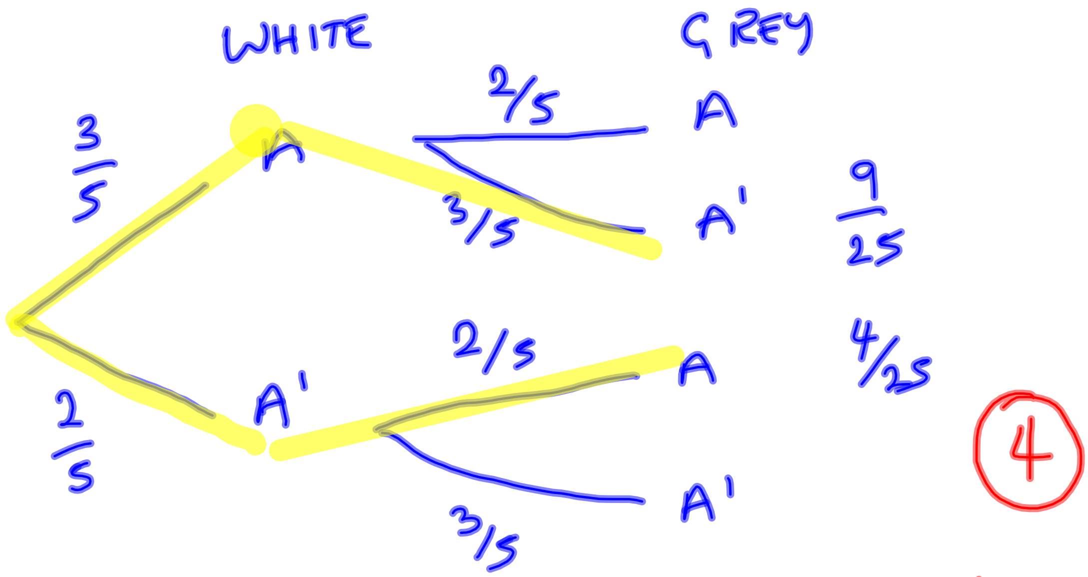
$= \frac{\overset{3}{\cancel{9}} \times \overset{4}{\cancel{8}} \times 7}{\cancel{3} \times \cancel{2} \times \cancel{1}}$

$= 12 \times 7$

$= \underline{84}$

P(A from grey)

$= \frac{84}{210} = \frac{42}{105} = \frac{14}{35} = \frac{2}{5}$



$$P(\text{exactly } 1\ A) = \frac{4}{25} + \frac{9}{25} = \frac{13}{25}$$

7 The probability distribution of a discrete random variable, X, is shown below.

x	0	2
$P(X = x)$	a	$1 - a$
px	0	$2(1-a)$

(i) Find $E(X)$ in terms of a. [2]

$$\sum px = 2(1-a)$$
$$= 2 - 2a$$

2

(ii) Show that $\text{Var}(X) = 4a(1 - a)$. [3]

x	0	2
x^2	0	4
$P(X=x)$	a	$1-a$
px^2	0	$4(1-a)$

$$\sigma^2 = \sum x^2 p - \mu^2$$
$$= 4(1-a) - (2-2a)^2$$
$$= 4 - 4a - (4 - 8a + 4a^2)$$
$$= \cancel{4} - 4a - \cancel{4} + 8a - 4a^2$$
$$= 4a - 4a^2$$
$$= 4a(1-a)$$

$(2-2a)(2-2a)$

F 4
O $-4a$
I $-4a$
L $+4a^2$
S $4 - 8a + 4a^2$

3

8 Five dogs, A, B, C, D and E, took part in three races. The order in which they finished the first race was $ABCDE$.

(i) Spearman's rank correlation coefficient between the orders for the 5 dogs in the first two races was found to be -1. Write down the order in which the dogs finished the second race. **[1]**

EDCBA ✓ 1

(ii) Spearman's rank correlation coefficient between the orders for the 5 dogs in the first race and the third race was found to be 0.9.

(a) Show that, in the usual notation (as in the List of Formulae), $\Sigma d^2 = 2$. **[2]**

$$r_s = 1 - \frac{6\sum d^2}{n(n^2-1)}$$

$$1 - \frac{6\sum d^2}{n(n^2-1)} = 0.9$$

$$1 - \frac{6\sum d^2}{5(24)} = 0.9$$

$$1 - \frac{\cancel{6}\sum d^2}{\cancel{120}\ 20} = 0.9$$

$$1 - \frac{\sum d^2}{20} = 0.9$$

$$20 - \sum d^2 = 18$$

$$\sum d^2 = 20 - 18$$

$$\underline{\underline{\sum d^2 = 2}}$$ ✓ 2

(b) Hence or otherwise find a possible order in which the dogs could have finished the third race. **[2]**

$\sum d^2 = 2$

RACE 1	RACE 3	RANK 1	RANK 2	d	d^2
A	B	1	2	−1	1
B	A	2	1	1	1
C	C	3	3	0	0
D	D	4	4	0	0
E	E	5	5	0	0
					2

BACDE ✓

MS: or equivalent (ie one placing reversed)

2